



ontario hydro research division

COMPLIMENTARY COPY

Please Credit Ontario Hydro Research if
Reproduced in Whole or in Part

HOT WATER SYSTEM DESIGN FOR MULTI-RESIDENTIAL BUILDINGS

Report No. 87-239-K

T.C. Olivares
Mathematician
Reliability & Statistics Section
Operations Research Department

ABSTRACT

Monitoring of hot water consumption was done for five multi-residential buildings. The hot water consumption was analyzed with the purpose of developing a design criterion for future domestic hot water systems. It was found that hot water consumption during weekends, especially winter weekends was significantly heavier than during other times. Two methods, involving the Cumulative and the Spielvogel design curves, were used to arrive at an optimal storage capacity and recovery rate combination. The computations were based on the winter weekend consumption. It was found that in most cases, the results derived from both methods were the same, and because of its simplicity, the use of the Cumulative design curve is recommended.

job	72-135-500	file	809.7	date	December 16, 1987	report no.	87-239-K
-----	------------	------	-------	------	-------------------	------------	----------

TABLE OF CONTENTS

	<u>Page</u>
1. INTRODUCTION	1
2. THE STATISTICAL ANALYSIS	1
3. DESCRIPTION OF DESIGN METHODS	5
4. CONCLUSIONS	6
5. RECOMMENDATIONS	7
REFERENCES	8
FIGURES	9 - 29
APPENDIX A: Comparison of Weekday and Weekend Hot Water Consumption	30
APPENDIX B: Cumulative and Spielvogel Design Curve Algorithms	31 - 35
DISTRIBUTION	last page



ontario hydro
research division

To Dr. D. Mills
Director of Research

HOT WATER SYSTEM DESIGN FOR MULTI-RESIDENTIAL BUILDINGS

1. INTRODUCTION

One of the major concerns of utilization engineers today is the design of domestic hot water system capable of coping with the hot water demand efficiently. The need for innovative improvements in developing efficient domestic hot water systems in multi-residential buildings has been recognized in Ontario Hydro where a large scale sampling plan was implemented to monitor hot water and energy consumption in a senior citizen building, three condominiums and a rental building. The main objective of the present study was to come up with an efficient design method for domestic hot water systems based on the information gathered from the five buildings. This study was requested by the Utilization Section of the Electrical Research Department.

Details on the monitoring of hot water consumption and a description of the preliminary results are contained in the references/1,2/.

2. THE STATISTICAL ANALYSIS

For this study, the senior citizen building, the three condominiums and the rental building will be identified by A, B, C, D, and E respectively.

Recording of data began in July of 1985 and was maintained for one full year. The raw data obtained for each building included hot water, natural gas, and energy consumption recorded in pulses at 15 minute intervals. These were converted and digitized into one-hour intervals, then transformed into standard files for analysis using the Statistical Analysis System software. An hourly hot water consumption distribution was created based on the overall validated days: some days were excluded in the analysis because of a breakdown in the monitoring system in these buildings. Later, the hourly hot water consumption distribution for each building was divided into seasonal, weekday and weekend distributions.

Descriptive statistics were calculated from the hot water consumption on the validated days. These included the mean, the maximum consumption and the standard deviation. The mean consumption, however, does not provide a good estimate for the amount of hot water required in a day. A statistic introduced by Maver/3/ was found useful and was therefore calculated. This statistic denoted by D_p is called the maximum predicted daily hot water consumption, it estimates the amount of hot water required daily in order to satisfy the demand with a certain degree of confidence. D_p is defined as:

$$D_p = d_p + \frac{v_q S}{n^{1/2}} \left(1 + \frac{v_p^2}{2}\right)^{1/2}$$

where:

$d_p = \bar{X} + v_p S$, is the amount of hot water to be made available,
daily that will satisfy the demand (1-p)100% of the days,

$\bar{X}$ is the mean daily hot water consumption,

S is the sample standard deviation of the daily hot water consumption,

p is the probability that consumption in any given day from a
collection of days will be greater in magnitude than d_p ,

q is the probability that demand can be met on at least (1-p)100% of the days,

v_p and v_q are values of the standard normal variates associated with p and q,

n is the number of days in the study period.

D_p values for various values of p and for q = 0.99 were calculated for each building and are given in Table I along with the other statistics. In the discussions that follow, Building B will be used as reference for more detailed illustrations.

TABLE I
MAXIMUM PREDICTED DAILY CONSUMPTION FOR FIVE BUILDINGS
(q = 0.99)

Bldg	Smpl Size	Mean	Std Dev	Maximum	D_p		
					p=0.05	p=0.025	p=0.01
A	369	67.76	14.23	89.90	93.82	98.60	104.25
B	339	248.26	36.47	376.82	315.34	326.83	342.14
C	399	219.31	26.23	312.68	267.15	275.94	286.32
D	300	303.08	47.82	425.94	391.61	407.80	426.90
E	136	396.22	39.32	501.28	472.95	486.71	502.97

As shown in Table I, the mean daily hot water consumption for Building B was 248.26 litres/suite with a maximum of 376.82 litres/suite. The corresponding D_p value with p = 0.05 and q = 0.99 was 315.34 litres/suite. This suggests that if 315.34 litres/suite of hot water is provided daily, the probability that the demand is met on at least 95% of the time or 347 days in a year is 0.99. For p = 0.025, the probability is 0.99 that the hot water supply of 326.83 litres/suite will meet the daily hot water demand on at least 97.5% of the time or 356 days in a year.

The D_p values for building A are somewhat unusual. It appears that, while in 99% of the years the number of days when the demand exceeds the daily supply of 93.82 litres/suite is 18 or less, in the given year there were no such days at all: the yearly maximum demand was only 89.9 litres/suite. Either this had to be a very unlikely year or the estimation method is not fully applicable because of the possibly unusual pattern of hot water consumption in a senior citizen building.

From previous results/2/, it was observed that the hot water usage during winter was heavier compared to other seasons and also weekend usage was generally heavier compared to that of weekdays. This indicates the presence of seasonal, weekday and weekend differences. Therefore further calculations were made on seasonal, weekend and weekday distributions. The results are given in Tables II, III and IV.

It was observed that the weekend consumption was relatively higher than the weekday consumption. To verify this, a test for a difference in means was carried out (See appendix A). Except for building A, the rest of the buildings showed statistically significant difference between the weekday and weekend consumption at 1% level of significance. For example, in Tables II and III, Building B has a weekday and weekend hot water consumption of 239.89 litres/suite and 268.54 litres/suite, respectively. Based on the test for comparison of means, the weekend consumption is significantly larger than the weekday hot water consumption with 99% confidence. From Table V it is clear that, the winter weekend hot water consumption is larger for each building than the overall, weekday or weekend hot water consumptions.

The hourly hot water consumption patterns were constructed and are shown in Figures 1-5. It is observed that the winter weekend curves are more asymmetric and have steeper slopes compared to the overall and weekend consumption patterns. The cumulative hot water consumption patterns in Figures 6-10 show that the winter weekend curve intersects the two other curves, at about the middle of the day. This reflects generally steeper slopes at each interval hours for winter weekends or a heavy "rise and fall" in the hot water usage. It was therefore decided, that the winter weekend consumption pattern will be the most suitable to use for coming up with a design criterion for the domestic hot water system.

TABLE II
MAXIMUM PREDICTED WEEKDAY HOT WATER CONSUMPTION
($q = 0.99$)

Bldg	Smpl Size	Mean	Std Dev	Maximum	D_p		
					p=0.05	p=0.025	p=0.01
A	263	66.87	14.09	89.90	94.39	99.19	104.84
B	240	239.89	29.01	357.86	294.30	304.21	315.89
C	283	214.59	22.71	281.16	256.77	264.48	273.57
D	214	291.79	45.45	425.94	377.65	393.24	411.63

TABLE III
MAXIMUM PREDICTED WEEKEND HOT WATER CONSUMPTION
(q = 0.99)

Bldg	Smpl Size	Mean	Std Dev	Maximum	D _p		
					p=0.05	p=0.025	p=0.01
A	106	66.67	14.62	87.55	95.99	101.17	107.30
B	99	268.54	44.14	376.82	357.02	372.73	391.32
C	116	230.83	30.45	312.68	291.02	301.77	314.47
D	86	331.16	41.84	422.37	416.11	431.13	448.90
E	37	433.9	32.63	501.28	506.75	519.22	534.02

TABLE IV
MAXIMUM PREDICTED WINTER WEEKEND HOT WATER CONSUMPTION
(q = 0.99)

Bldg	Smpl Size	Mean	Std Dev	Maximum	D _p		
					p=0.05	p=0.025	p=0.01
A	21	73.55	5.65	85.20	87.52	89.80	92.52
B	21	304.57	18.48	345.12	349.37	356.84	365.73
C	25	248.91	25.46	312.68	309.00	319.10	331.11
D	25	354.16	24.53	422.37	412.04	421.77	433.34
E	3	479.6	28.57	501.28	585.55	601.28	620.24

3. DESCRIPTION OF DESIGN METHODS

The domestic hot water system as a physical system is composed of two major components, the boiler and the storage tank. These components are represented in the models by the recovery rate of hot water and the storage capacity, respectively.

Maver/3/ describes a method of determining a set of optimal storage and recovery combinations. Its product is an optimal curve called the Cumulative design curve. The set of points on the curve correspond to the set of optimal combination for storage capacity and recovery rate. The curve is derived from the cumulative hourly hot water consumption pattern. The algorithm works by maintaining a minimum recovery rate equal to the total daily hot water consumption divided by 24. A series of recovery values are generated by adding to the minimum recovery rate an increment equal to the difference between the maximum of the hourly consumption and the minimum recovery rate divided by 24. Storage capacity is derived by taking the difference between the recovery rate and the cumulative consumption at each hour when recovery rate is less than or equal to the cumulative hot water consumption, otherwise recovery rate is made equal to the cumulative hot water consumption. The algorithm generates twenty four points which correspond to the recovery rate and storage capacity values. When plotted, these twenty four points comprise the Cumulative design curve.

To accommodate the variability of the hot water consumption within each hour, an upper 95% confidence interval was constructed on the winter weekend hot water consumption distribution for each building. Figures 11-15 show the Cumulative design curves based on the upper 95% winter weekend confidence interval and the overall distribution. For example, Building B has 20 litres/hour/suite of recovery rate and 33 litres/suite of storage capacity as one of the optimal combinations from the upper 95% winter weekend confidence interval. For the same amount of recovery rate, the storage capacity recommended by the overall distribution is only zero.

Another method of constructing a design curve was introduced by Werden and Spielvogel/4/. The algorithm starts by determining the one-hour peak, two-hour peak, three-hour peak, etc. of the hot water consumption. The design curve's objective is to accommodate these peak demands. Storage capacity is the total hot water consumption of a given interval peak. Recovery on the other hand, is calculated by dividing the storage capacity of each interval peak by the number of hours. For example, the recovery rate corresponding to a five-hour interval peak equals the storage capacity of this period divided by five. Consider a recovery rate, R_i and a storage capacity, S_i for $i = 1, 2, \dots, 24$, associated with the i th-hour interval peak. Suppose now, a pairwise set of points is generated of the form $(R_i, 0)$ and $(0, S_i)$ for each i and plotted in a coordinate axis. If a straight line is drawn connecting the two points $(R_i, 0)$ and $(0, S_i)$ for each i , these will generate 24 lines like the illustration in Figure 21.

The Spielvogel curve is the envelope of the 24 lines. The storage capacity and recovery rate combinations given by the Spielvogel curve will satisfy any of the hour interval peak demands. Also, minimum recovery is set to equal the total consumption divided by twenty-four. When this value is reached, the algorithm maintains this value for larger storage capacity values.

Figures 16-20 show the Spielvogel design curves for Buildings A through E based on the upper 95% winter weekend confidence interval. Table V compares selected optimal combinations given by the Cumulative and the Spielvogel design curves.

TABLE V
SELECTED OPTIMAL STORAGE AND RECOVERY COMBINATION
 (Values are approximations based on the design curves)

Building	Recovery (litres/suite/hour)	Storage	
		Cumulative (litres/suite)	Spielvogel
A	5	7.3	7.1
B	20	33.0	33.0
C	16	32.0	31.0
D	20	56.0	56.0
E	34	52.5	54.0

The results in the Table II show that the values for both design curves are very close except for Building E. The results for Building E however, must be considered with caution because of the small sample of three winter weekends only.

The values for the Cumulative and Spielvogel curves were generated using a BASIC program prepared by Mr. N. Milligan, Utilization Section, Electrical Research Department. The listings of both programs are attached in Appendix B.

4. CONCLUSIONS

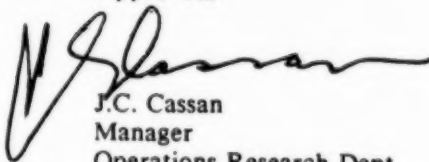
- Except for building A, the rest of the buildings displayed statistically significant differences between the weekend and the weekday consumption with 99% confidence.
- A comparison of the mean hot water consumption given in Tables I to IV disclosed heavier consumption on winter weekends for all the buildings.
- The hourly hot water consumption frequency plots show that on weekends, peak demand is an hour or two later in the morning than on weekdays, and winter weekends have their peak demand even more towards the middle of the day.
- The cumulative hourly hot water pattern based on the winter weekend consumption for each building rises above the overall and weekend consumption patterns towards the middle of the day and maintains the higher position for the rest of the day. This implies heavy fluctuations in hot water use and heavier hot water consumption during the winter weekends. However, results from building E should be used with caution because of the small sample size of only three winter weekends.
- The winter weekend hot water consumption distribution was used in constructing the design curves. The design combinations for recovery rate and storage capacity recommended by the Spielvogel design curve were close to that given by the Cumulative design

curve.

5. RECOMMENDATIONS

- To determine the best storage capacity and recovery rate combination, it is recommended that a cost function for each design curve be established. The cost function should include costs for the storage tank, boiler and other physical systems involved in generating hot water. With the cost function known, this can be plotted together with the design curve. The point where both curves touch corresponds to the recovery rate and storage capacity combination that minimizes the cost function. This is discussed further in Maver's paper/3/.
- For future applications, it is recommended that the Cumulative design curve algorithm be used because of its simplicity and flexibility.

Approved:



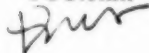
J.C. Cassan
Manager
Operations Research Dept

Submitted:



T.C. Olivares
Mathematician
Reliability & Statistics Section

TCO:cmh



REFERENCES

1. Ciz, J.B., "Performance of Domestic Hot Water Systems in Five Apartment Buildings. Part I - Installation and Commissioning", Ontario Hydro Research Report 86-77-H, 1986.
2. Milligan, N.H., "Performance of Domestic Hot Water Systems in Five Apartment Buildings. Part II - Analysis and Results", Ontario Hydro Research Report 87-53-H, 1987.
3. Maver, T.W., "Some Techniques of Operations Research Illustrated by their Application to the Problem of Hot Water Plant Sizing", Journal of Institutional Heating and Ventilation Engineers, Vol. 33, No. 1, January 1966.
4. Werden, R.G. and Spielvogel L.G., "Sizing of Service Water heating Equipment in Commercial and Institutional Buildings. Part II - Research Program, Data Acquisition, Reduction and Analysis, Recommendations for Sizing", ASHRAE Transactions, Vol.75, Part 2, 1969.

FIGURE 1

HOURLY HOT WATER USE

BUILDING A

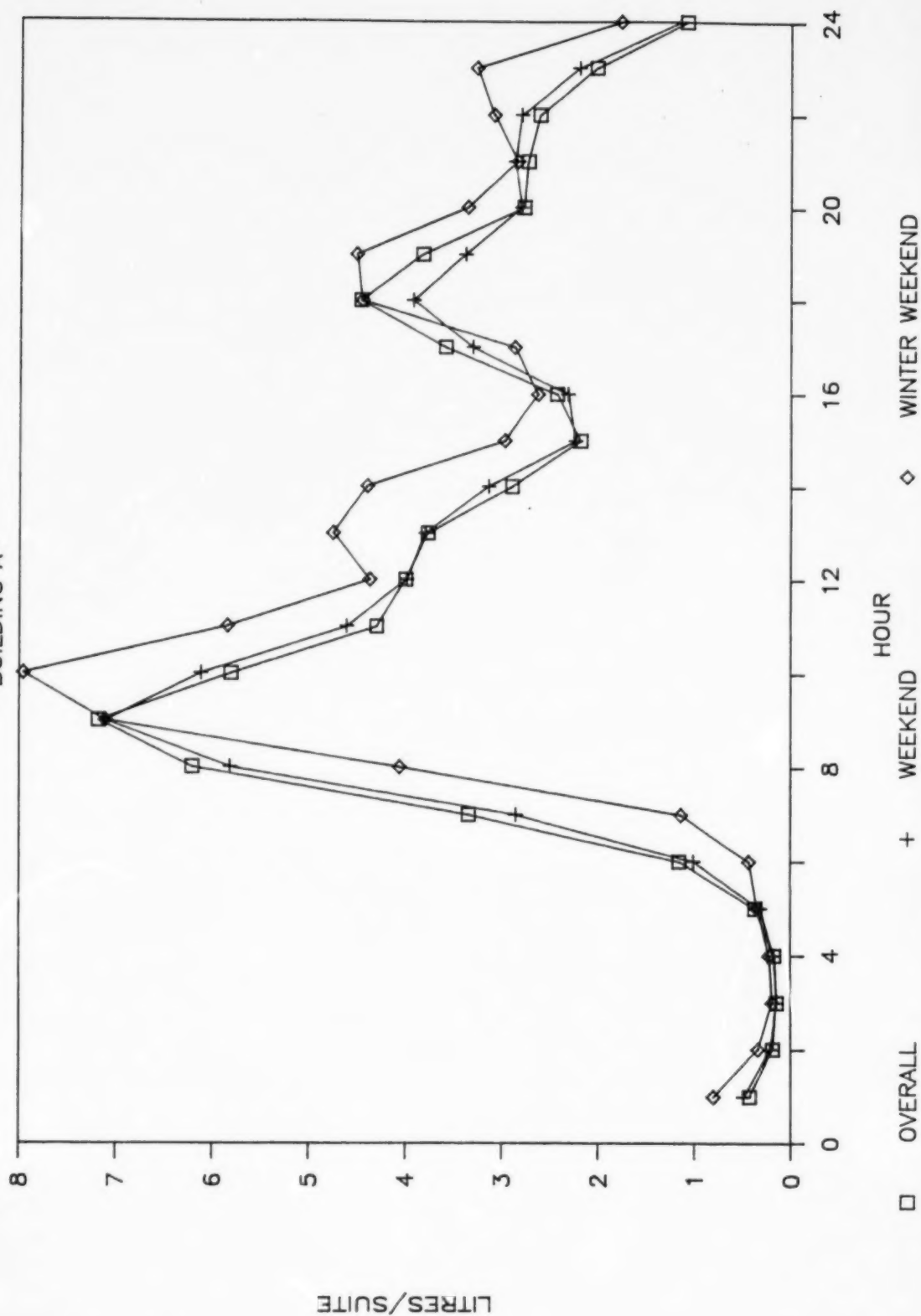


FIGURE 2

HOURLY HOT WATER USE

BUILDING B

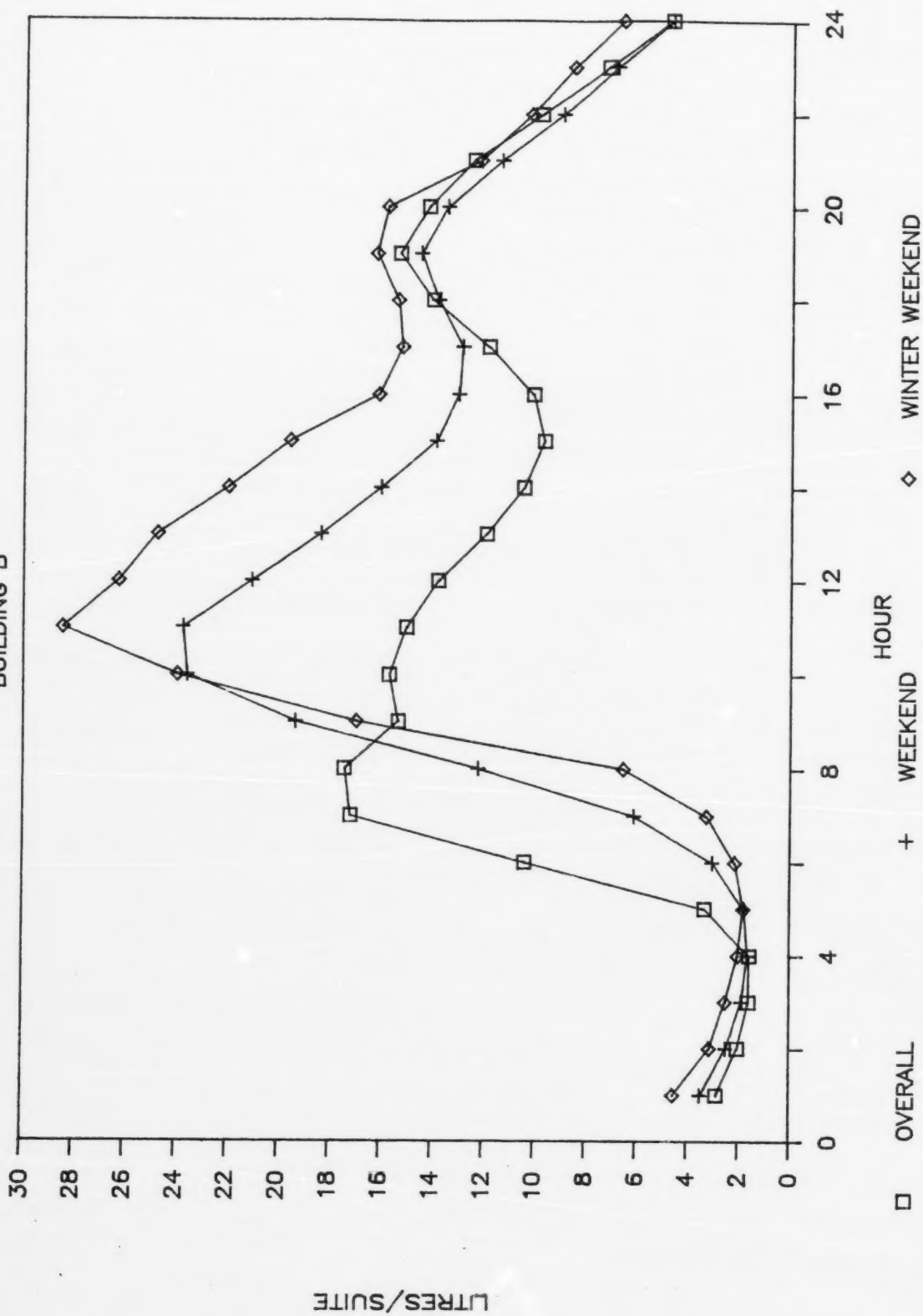


FIGURE 3

HOURLY HOT WATER USE

BUILDING C

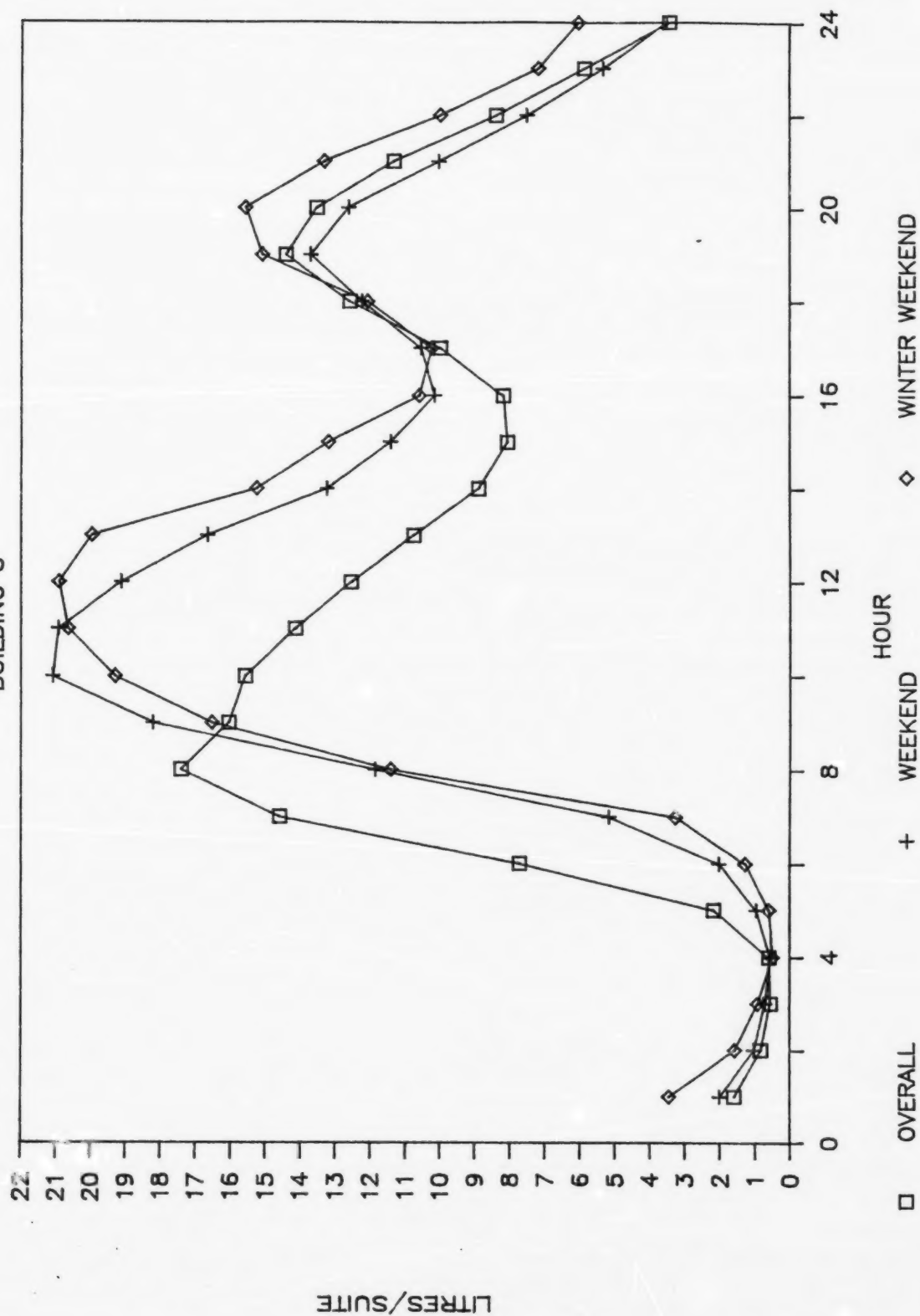


FIGURE 4

HOURLY HOT WATER USE

BUILDING D

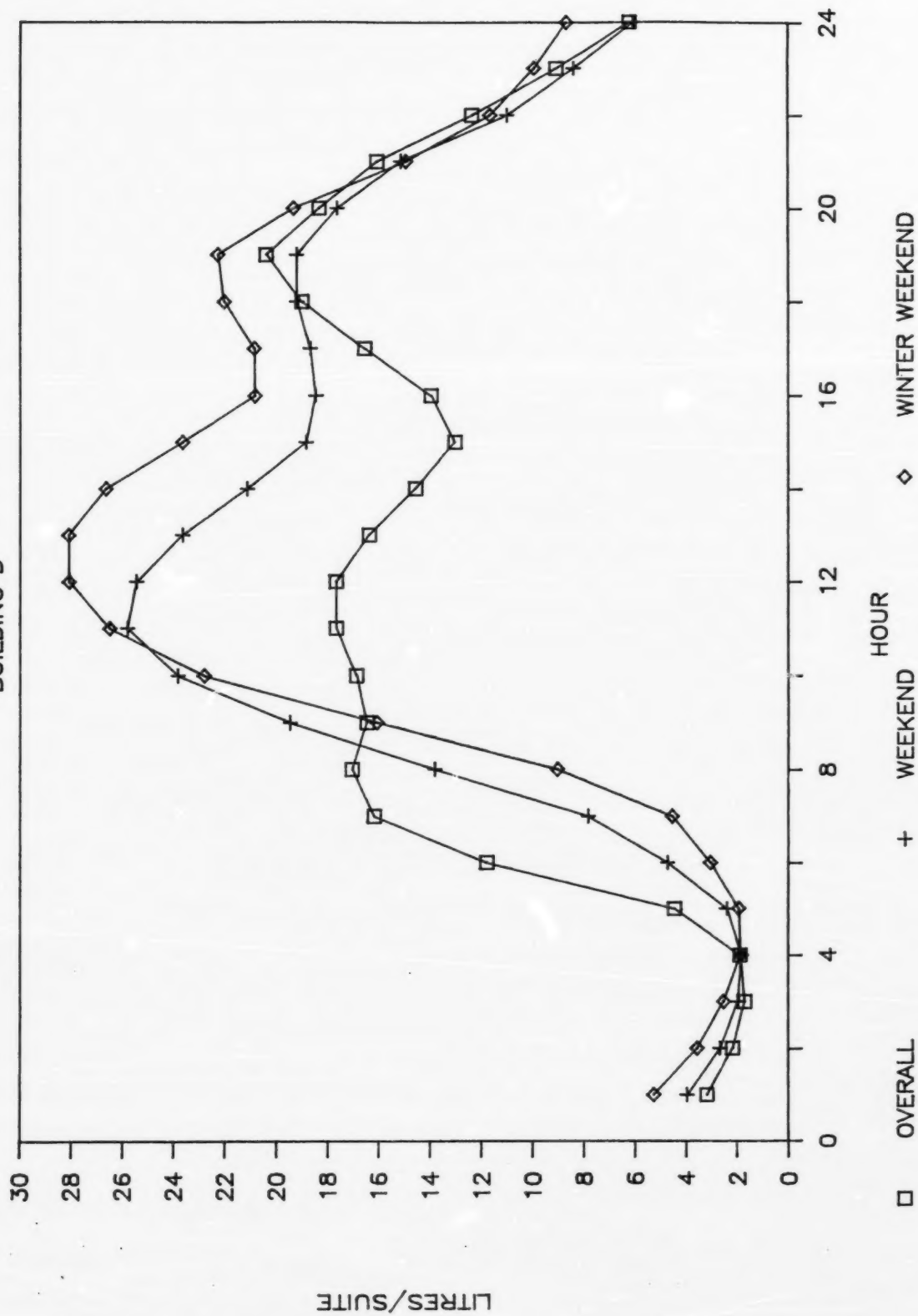


FIGURE 5

HOURLY HOT WATER USE

BUILDING E

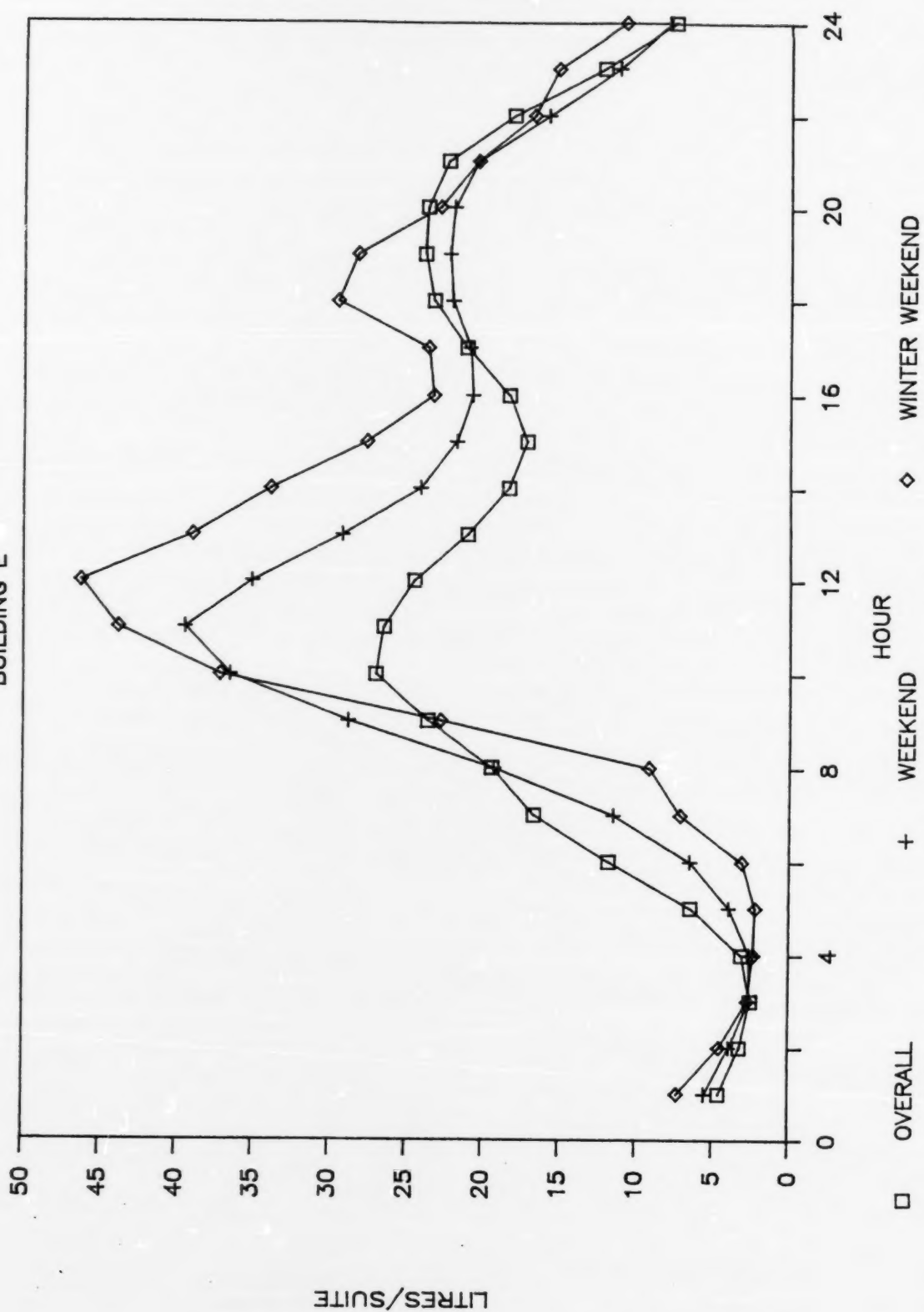


FIGURE 6

CUMULATIVE HOURLY HOT WATER USE

BUILDING A

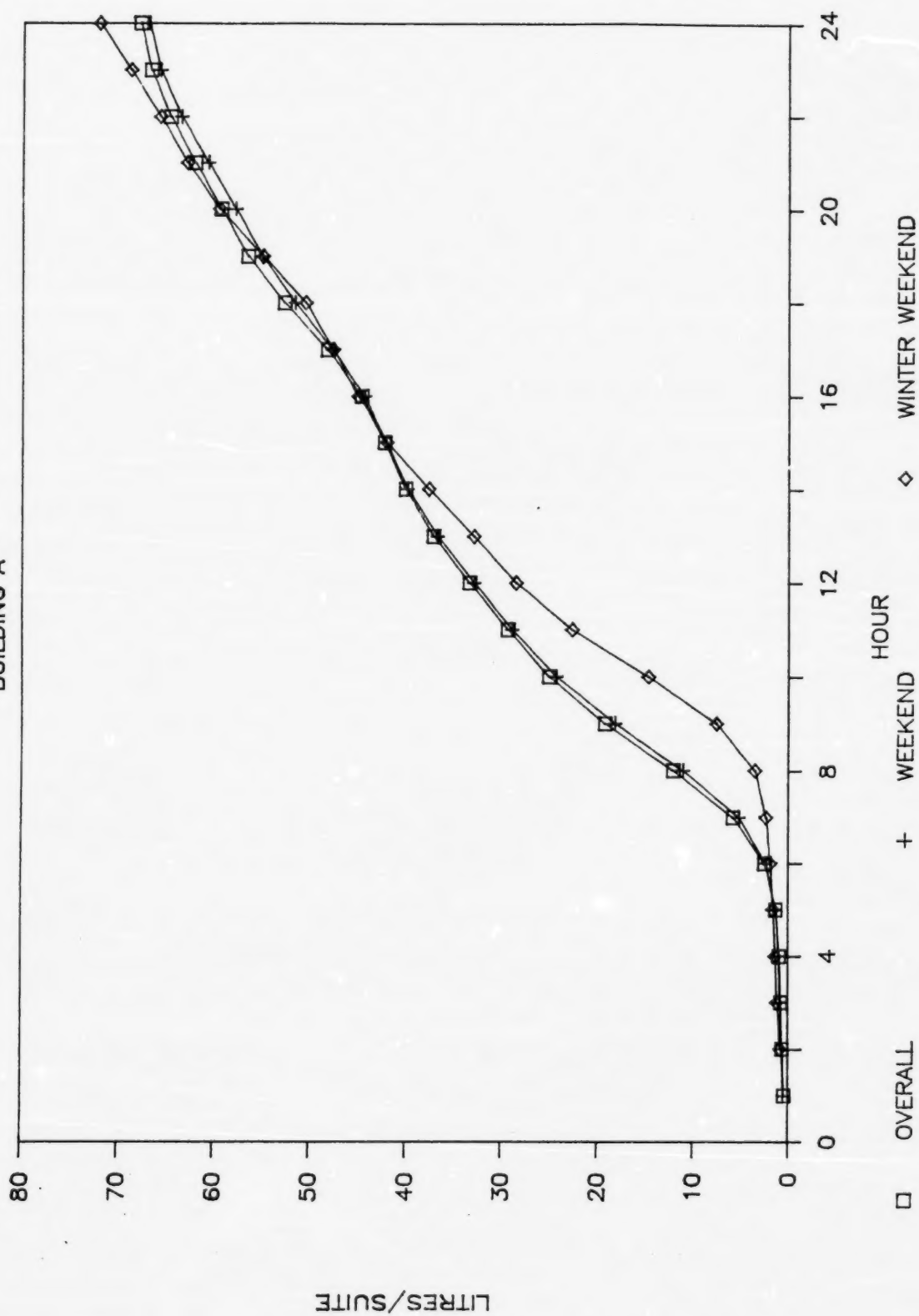


FIGURE 7

CUMULATIVE HOURLY HOT WATER USE

BUILDING B

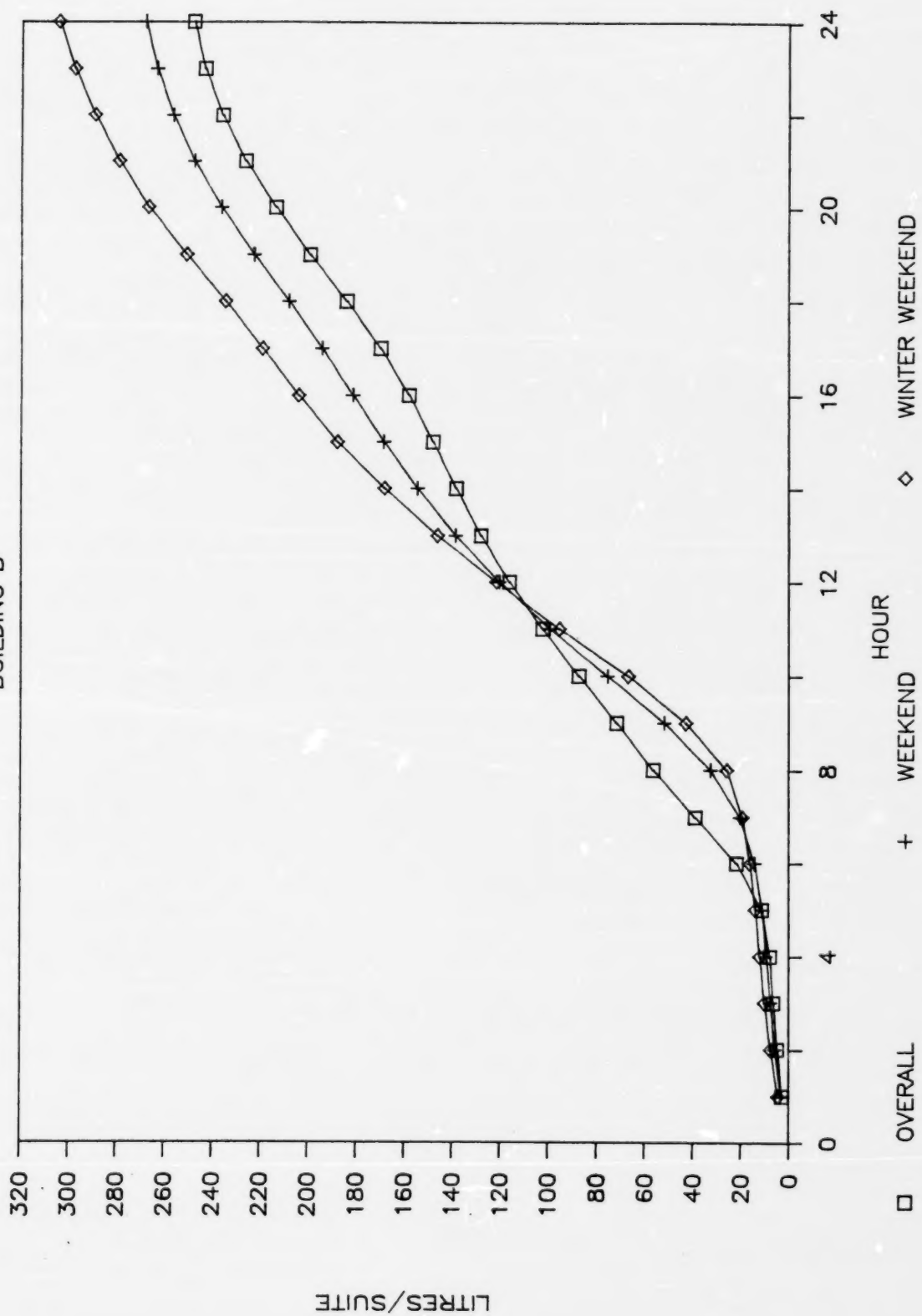


FIGURE 8

CUMULATIVE HOURLY HOT WATER USE

BUILDING C

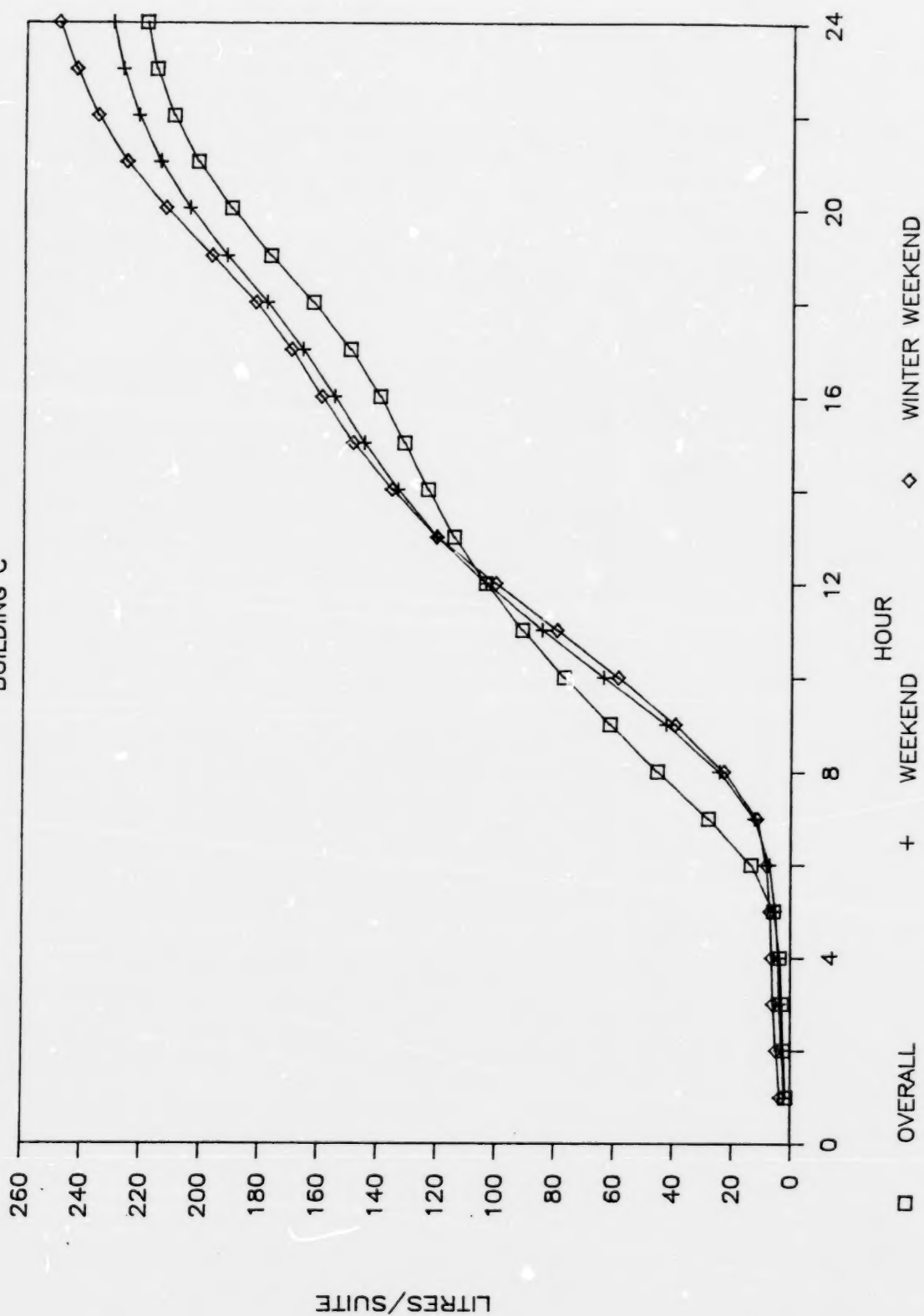


FIGURE 9

CUMULATIVE HOURLY HOT WATER USE

BUILDING D

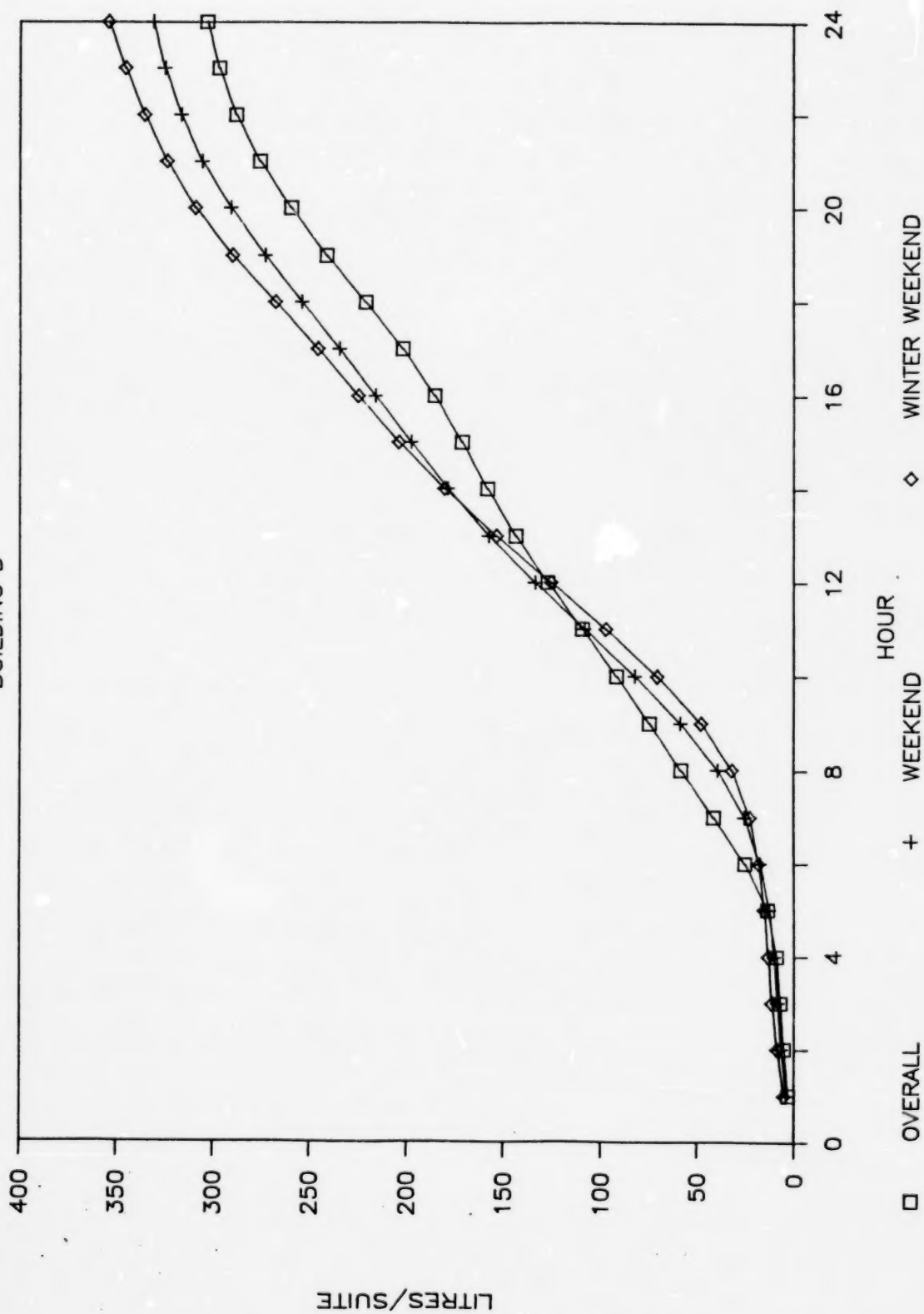


FIGURE 10

CUMULATIVE HOURLY HOT WATER USE

BUILDING E

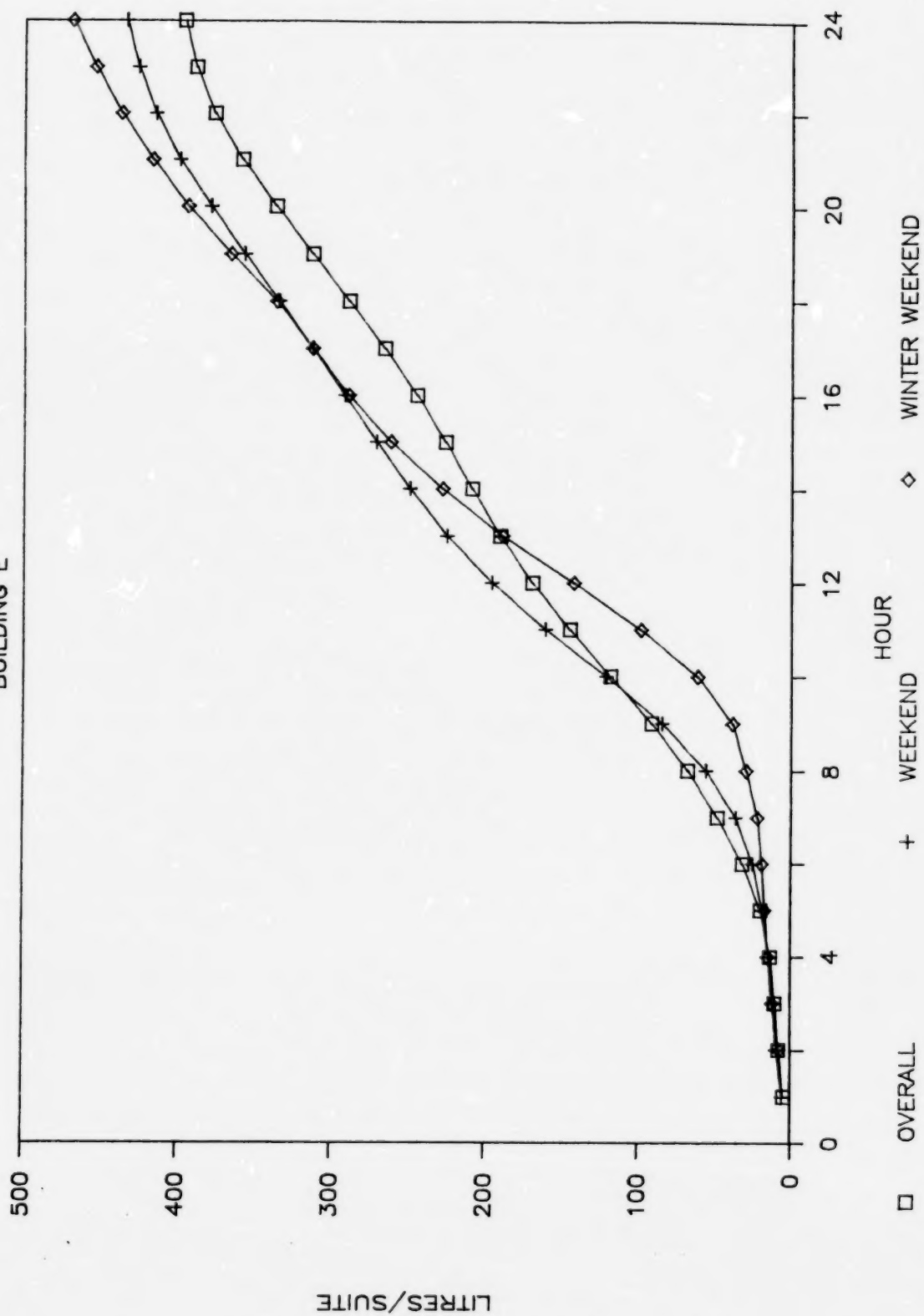


FIGURE 11

CUMULATIVE DESIGN CURVES

BUILDING A

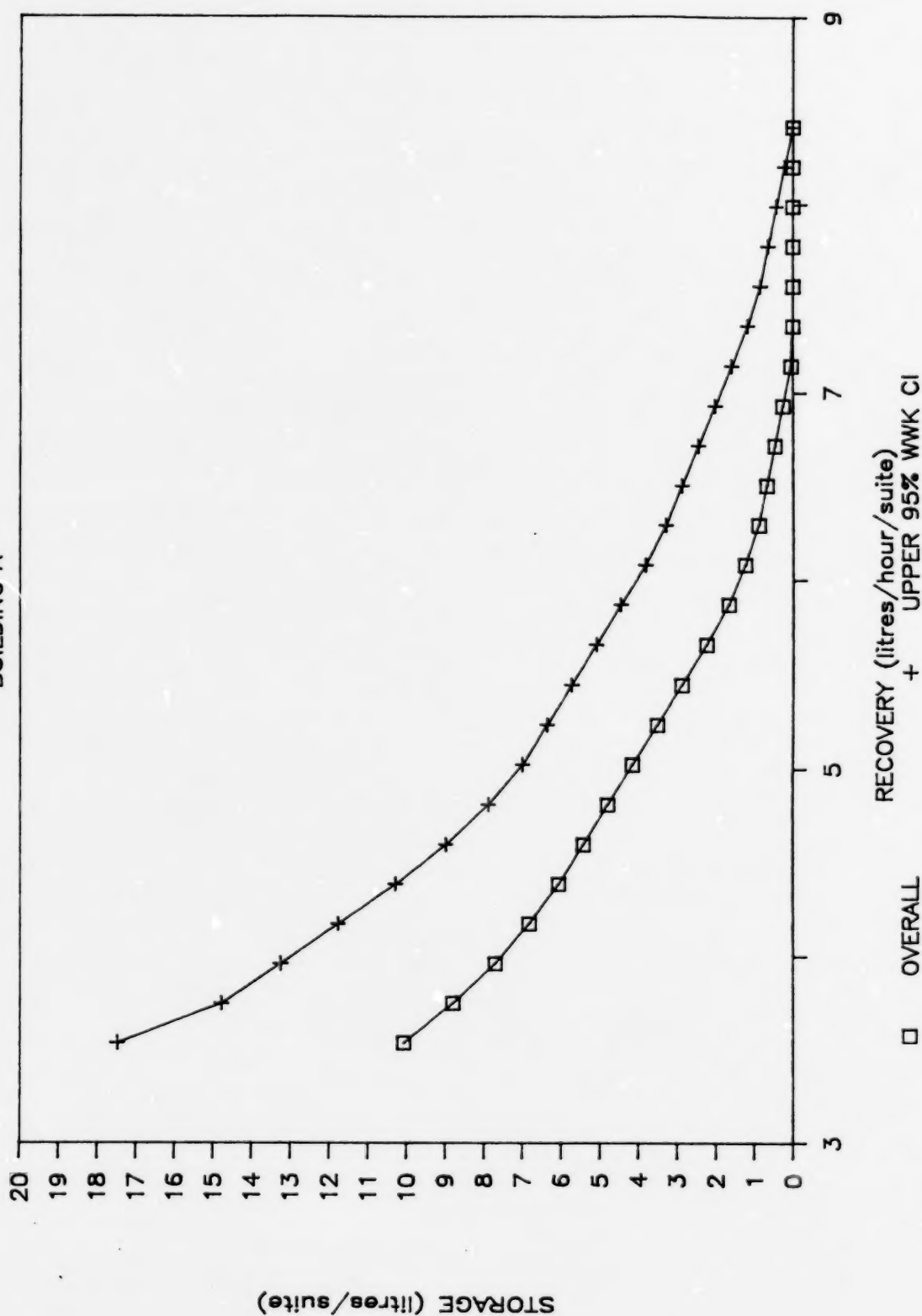


FIGURE 12

CUMULATIVE DESIGN CURVES BUILDING B

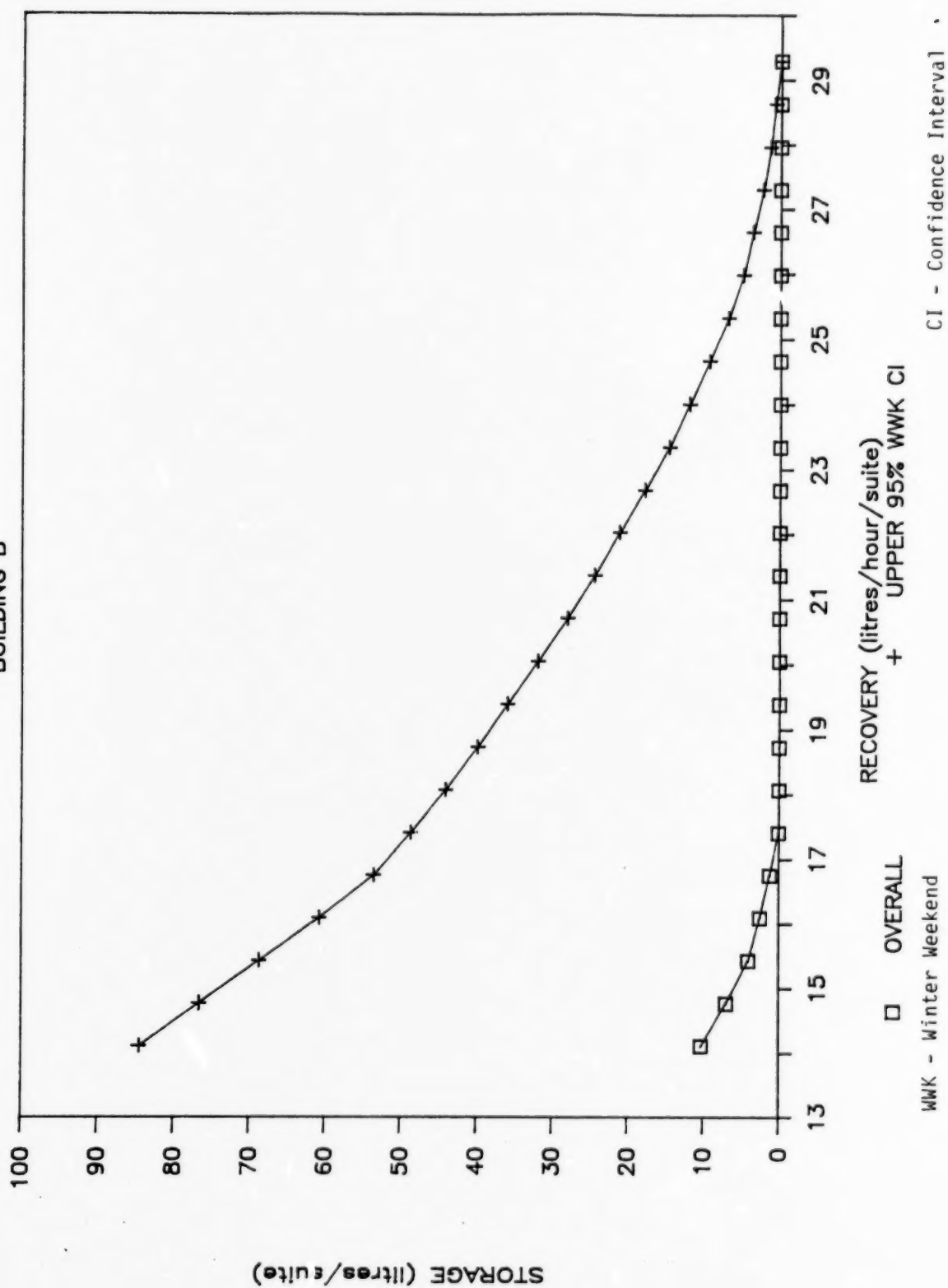


FIGURE 13

CUMULATIVE DESIGN CURVES

BUILDING C

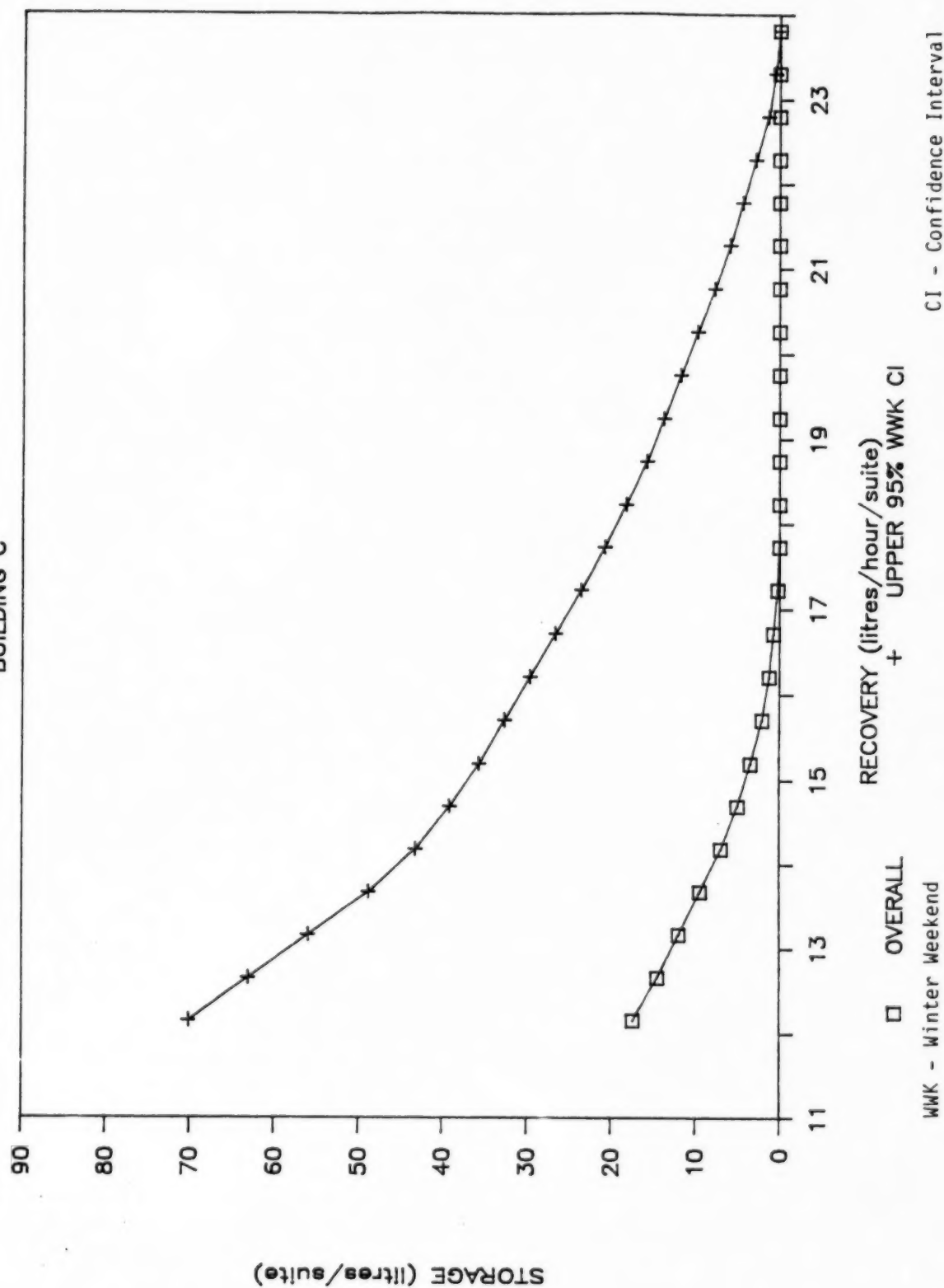
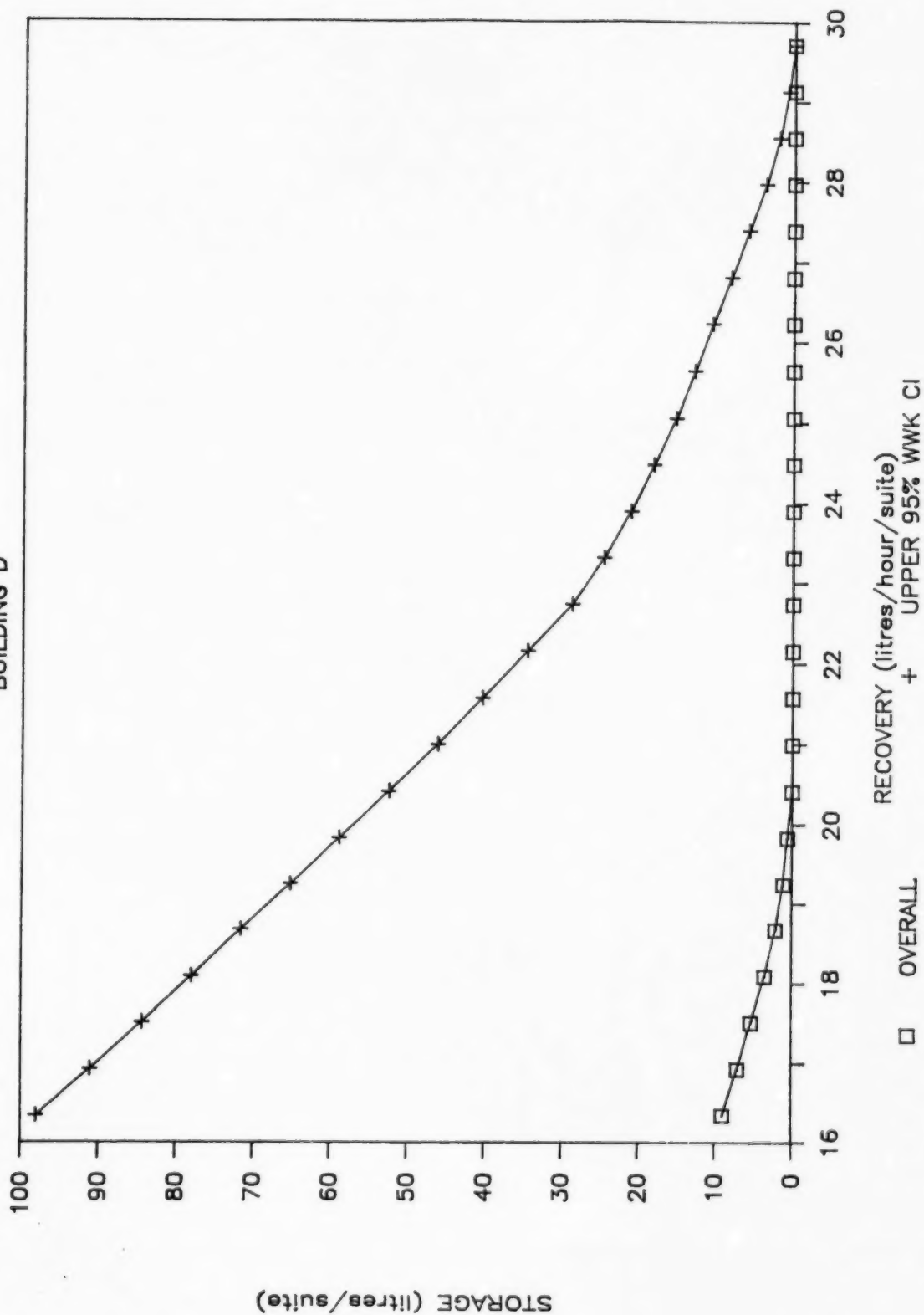


FIGURE 14

CUMULATIVE DESIGN CURVES

BUILDING D



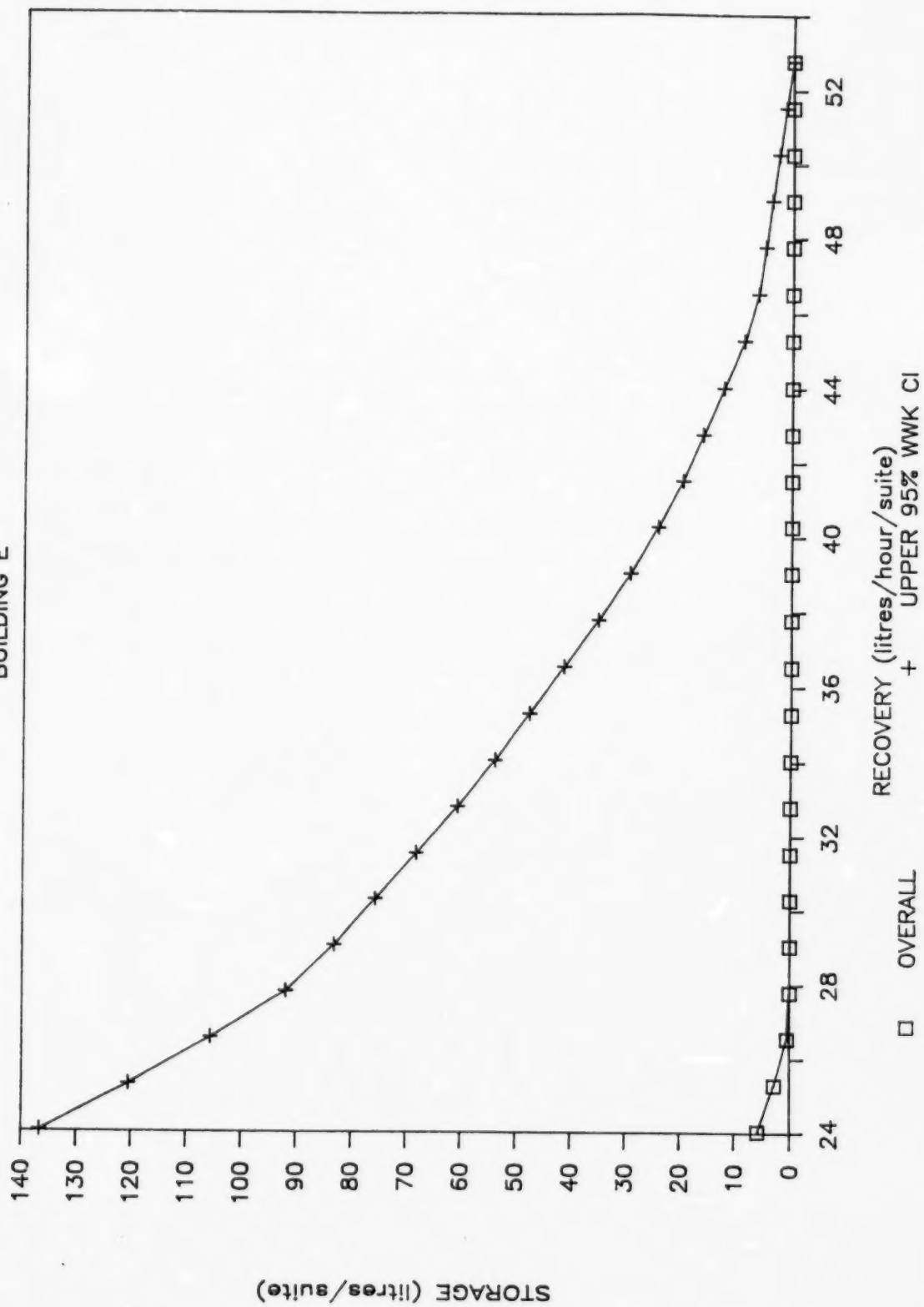
CI - Confidence Interval

WWK - Winter Weekend

FIGURE 15

CUMULATIVE DESIGN CURVES

BUILDING E



CI - Confidence Interval

WWK - Winter Weekend

FIGURE 16

SPIELVOGEL DESIGN CURVE

BUILDING A

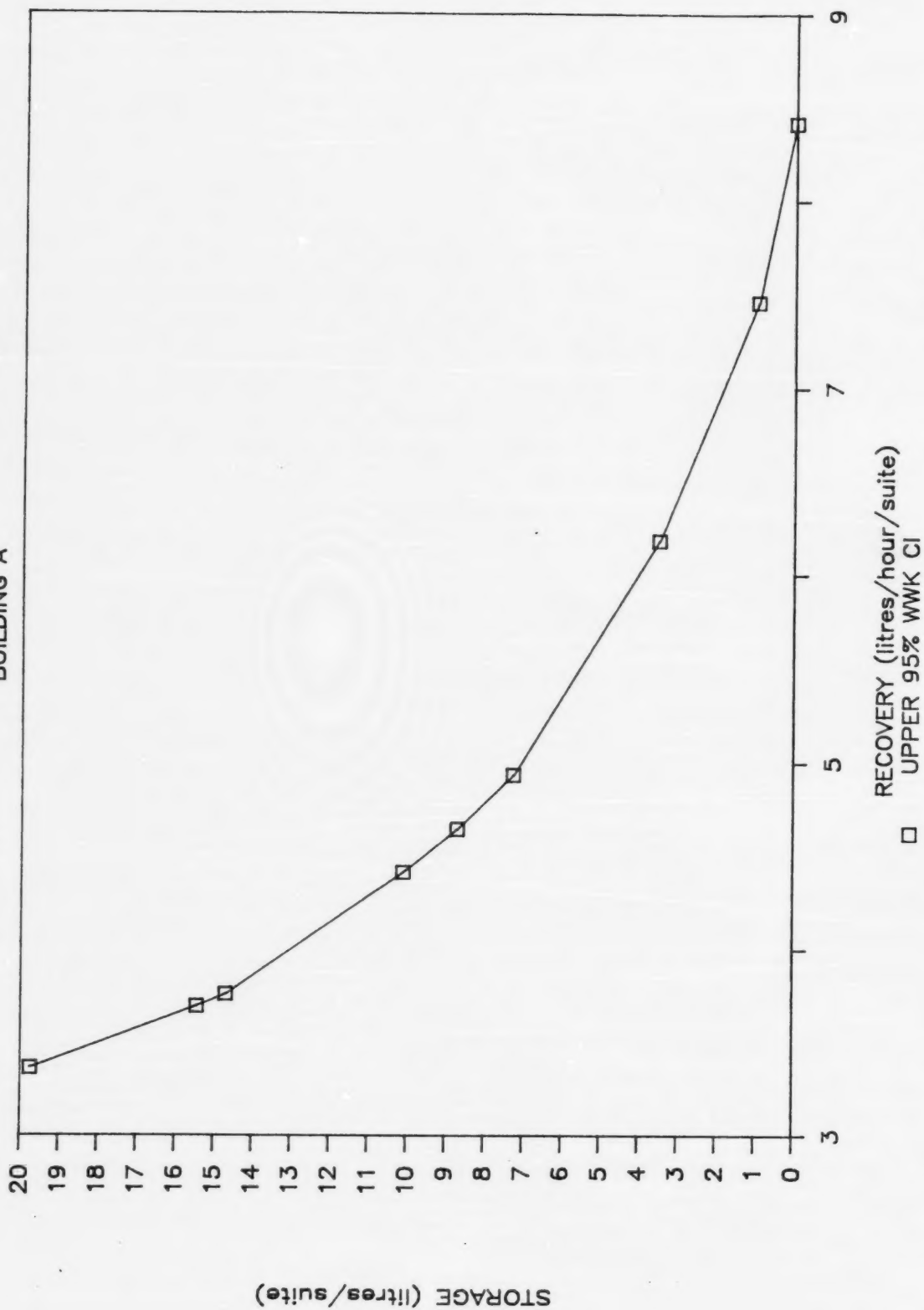


FIGURE 17

SPIELVOGEL DESIGN CURVE

BUILDING B

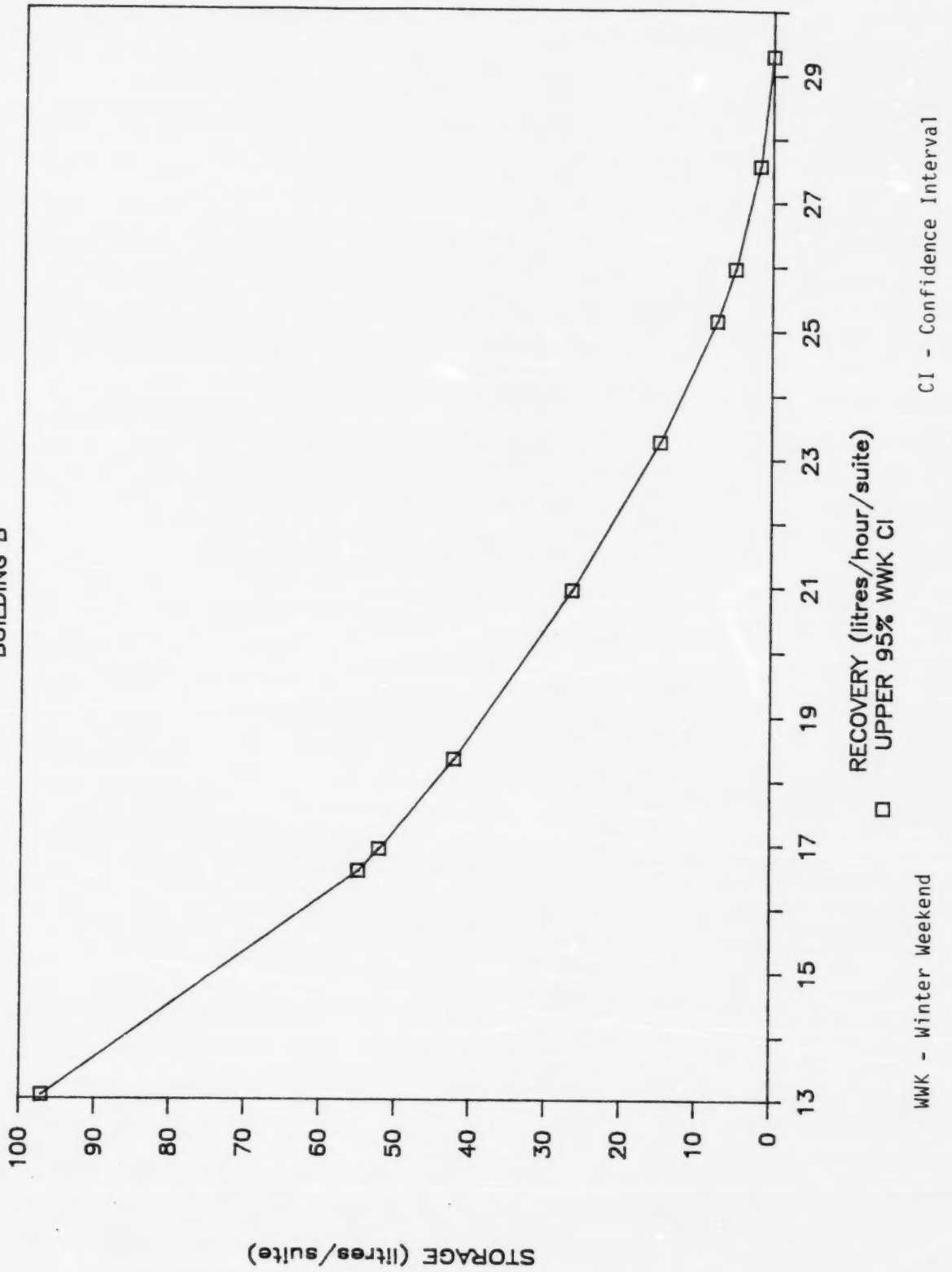


FIGURE 18

SPIELVOGEL DESIGN CURVE

BUILDING C

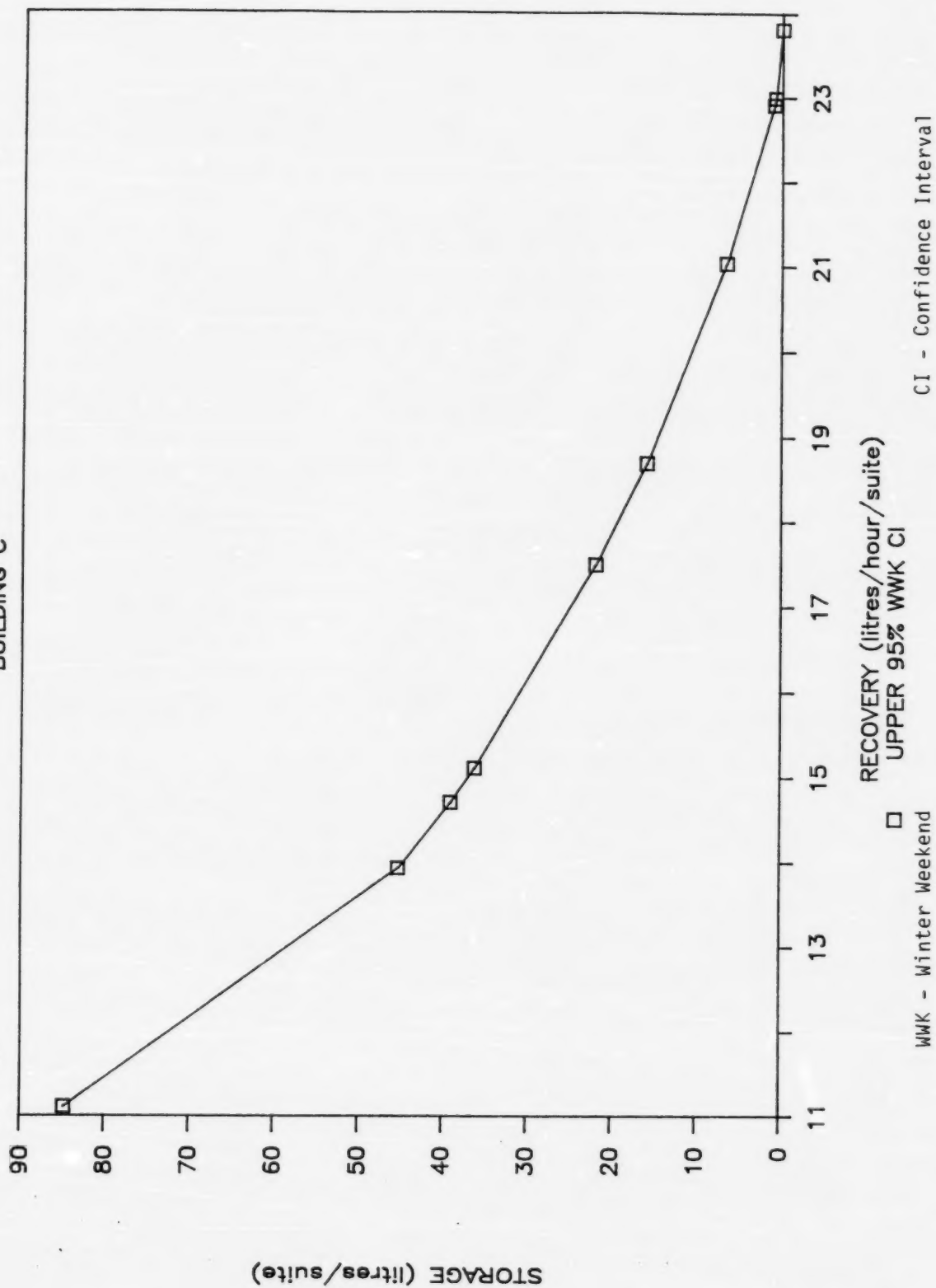


FIGURE 19

SPIELVOGEL DESIGN CURVE BUILDING D

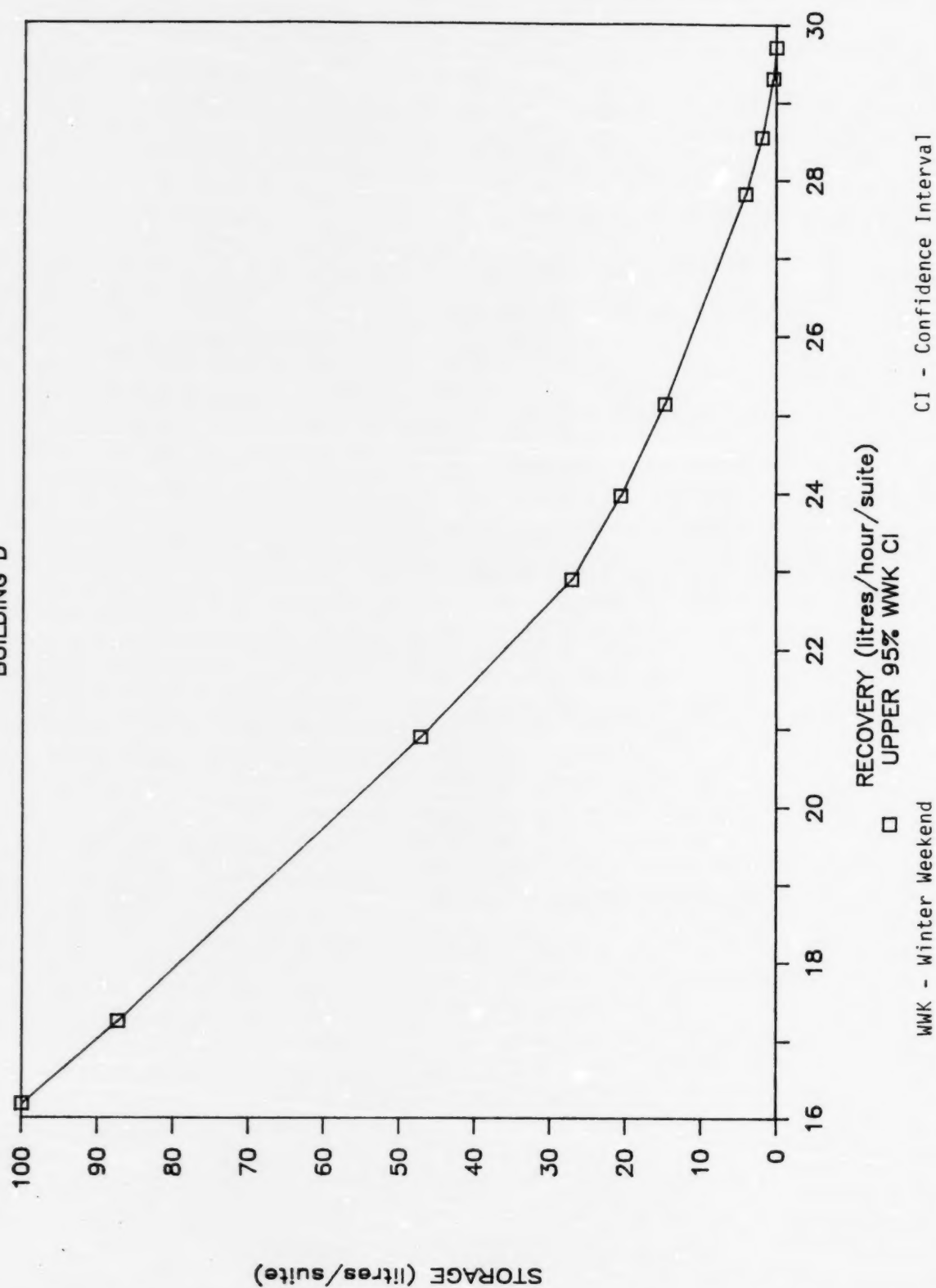
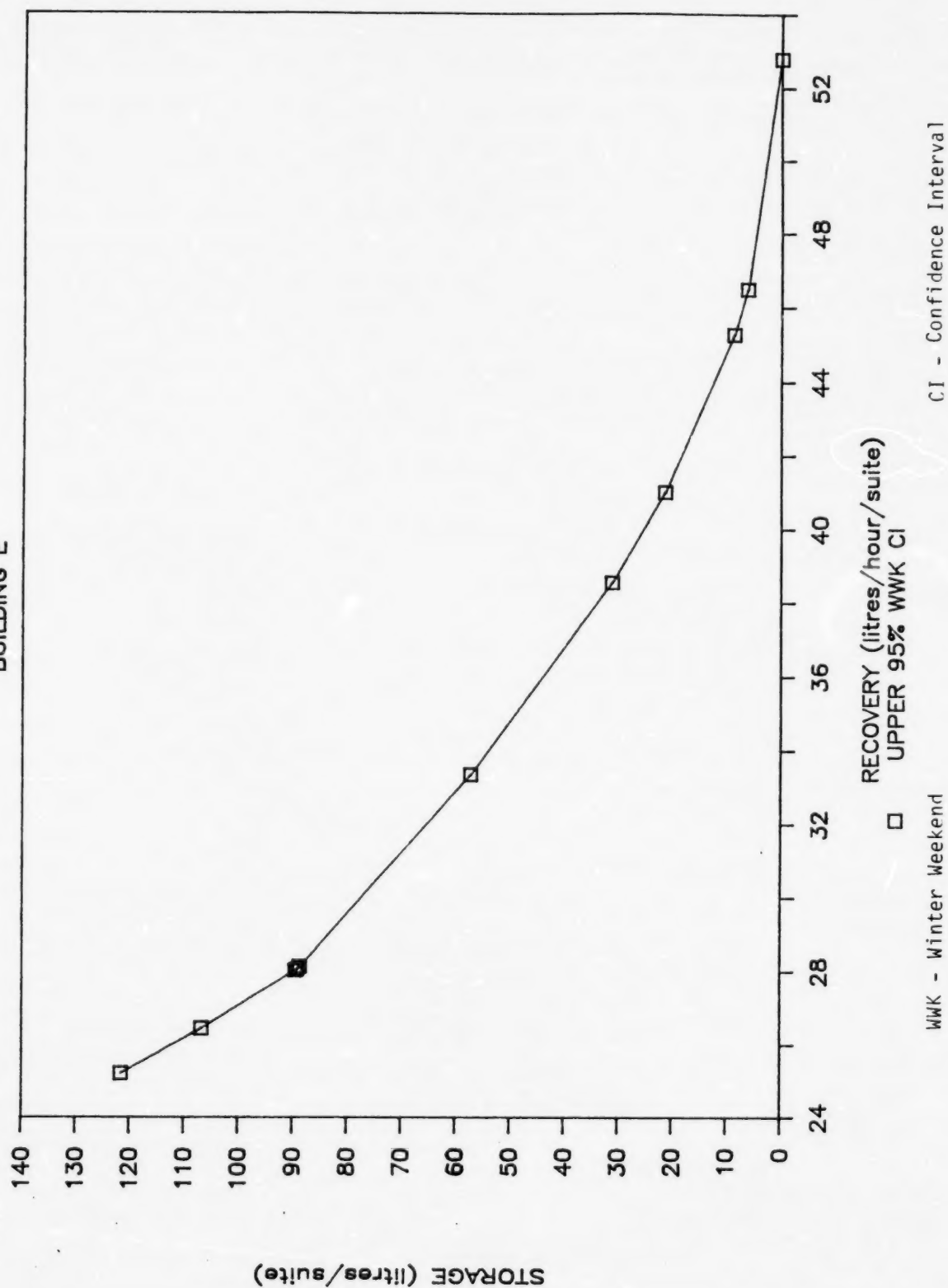


FIGURE 20

SPIELVOGEL DESIGN CURVE BUILDING E



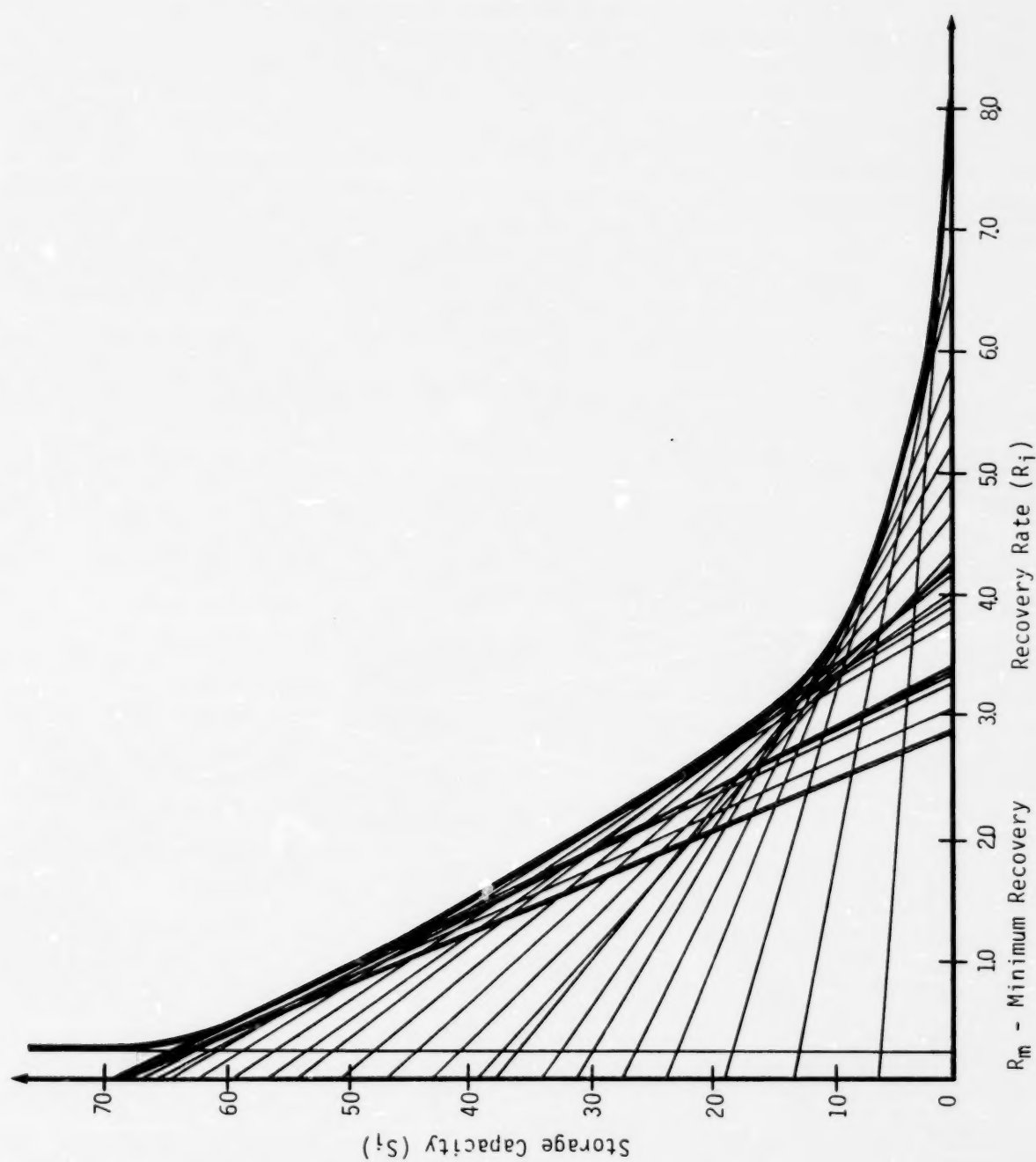


FIGURE 21
CONSTRUCTION OF A SPEILVOGEL CURVE

APPENDIX A

COMPARISON OF WEEKDAY AND WEEKEND HOT WATER CONSUMPTION

The following test for comparison of means indicates if there is a significant difference between the weekend and weekday hot water consumption of the five multi-residential buildings. The null hypothesis is defined as:

H_0 : There is no difference between the weekend and weekday hot water consumption, and the alternative hypothesis as:

H_1 : There is a difference between the weekend and weekday hot water consumption.

A summary of results for the test for difference of means is given below. Note that except for building A, the rest of the buildings displayed a significant difference between the weekend and weekday hot water consumption at a significance level of 0.01.

Building	Consumption		Z-Test	Decision
	Weekend	Weekday		
A	66.87	68.12	-0.75	Accept H_0
B	268.54	239.89	5.95	Reject H_0
C	230.83	214.59	5.18	Reject H_0
D	331.16	291.79	7.19	Reject H_0
E	433.90	382.13	8.32	Reject H_0

APPENDIX B

CUMULATIVE AND SPIELVOGEL DESIGN CURVE ALGORITHMS

```
10 REM program to calculate sizing curves using the cumulative
15 REM consumption method.
20 REM by: N.H. Milligan, 87-05-05
100 DIM A(24,5), B(24,5), C(24,5), D(24,5), R(24,5), RMAX(5), RMIN(5)
101 DIM N(5), A1(24,5), S1(24,5), S2(24,5), Y(24,5), Z(24,5)
110 CP=.001164: TEMP=60
150 REM read in hourly consumption data
160 CLS: LOCATE 5,2
165 INPUT "enter data file name to be read: ",F$
170 OPEN F$ FOR INPUT AS #1
175 FOR I=1 TO 5
180 INPUT #1,A$
185 FOR J=1 TO 24
190 INPUT #1, A1(J,I), B(J,I)
191 PRINT A1(J,I), B(J,I)
195 NEXT J
200 NEXT I
210 CLOSE #1
211 N(1)=106
212 N(2)=99
213 N(3)=116
214 N(4)=87
215 N(5)=37
216 FOR I= 1 TO 5
217 FOR J= 1 TO 24
218  $A(J,I)=A1(J,I)+((2.32 * B(J,I))/SQR(N(I)))$ 
219 PRINT A(J,I), B(J,I)
220 NEXT J
221 NEXT I
250 REM calculate cumulative consumption curves
260 REM read in winter weekend consumption data
265 CLS: LOCATE 5,2
270 INPUT "enter data file name to be read: ",F$
275 OPEN F$ FOR INPUT AS #2
285 FOR I=1 TO 5
286 FOR J= 1 TO 24
287 IF EOF(2) THEN 292
288 INPUT #2, Y(J,I), Z(J,I) :PRINT Y(J,I), Z(J,I)
289 NEXT J
290 NEXT I
```

APPENDIX B
(Cont'd)

```
291 CLOSE #2
292 FOR I=1 TO 5
293 FOR J= 1 TO 24
294 C(J,I) = C(J-1,I) + A(J,I)
295 D(J,I) = D(J-1,I) + Y(J,I)
297 NEXT J
298 NEXT I
300 REM find minimum recovery and maximum storage
310 FOR I=1 TO 5
320 GOSUB 1000: REM find maximum 1 hour recovery
325 RMIN(I)=C(24,I)/24
330 RINV=(RMAX(I)-RMIN(I))/24
335 FOR J=1 TO 24
337 R=0
340 RECV=RMIN(I)+RINV*J
345 FOR K=1 TO 24
350 R1=R1+RECV:R2=R2+RECV
360 IF R1=>C(K,I) THEN R1=C(K,I)
361 IF R2=>D(K,I) THEN R2= D(K,I)
365 DELTA1 = C(K,I) - R1
366 DELTA2 = D(K,I) - R2
367 IF DELTA1 > S1 THEN S1=DELTA1
368 IF DELTA2 > S2 THEN S2=DELTA2
370 NEXT K
374 S1(J,I)=S1
375 S2(J,I)=S2
380 R(J,I)=RECV
384 S1=0
385 S2=0
390 NEXT J
400 NEXT I
500 REM store storage and recovery in data file
510 LOCATE 10,2
520 INPUT" enter output file name: ";F$
530 OPEN F$ FOR OUTPUT AS #3
545 PRINT #3, 5
550 PRINT #3, "1"
555 FOR I = 1 TO 5
560 PRINT #3,24
570 FOR J=1 TO 24
575 WRITE #3, S1(J,I), S2(J,I), R(J,I)
578 PRINT S1(J,I), S2(J,I), R(J,I)
580 NEXT J
590 NEXT I
600 CLOSE #3
```

APPENDIX B
(Cont'd)

```
610 REM end of the program
620 END
1000 REM subroutine to find maximum one hour recovery
1010 FOR L=1 TO 5
1020 FOR M=1 TO 24
1030 IF A(M,L)>RMAX(L) THEN RMAX(L)=A(M,L)
1040 NEXT M
1050 NEXT L
1060 RETURN
```

```
5 REM program to generate spielvogel curves from apartment building
6 REM data.
7 REM By: N.H. Milligan, 87-05-04
10 DIM A(48,5), B(48,5), S(48,5), R(48,5), T(48,5), X(24,5), Y(24,5), X1(24,5), Y1(24,5)
20 CP=.001164: RHO=1!: THW=60
30 REM input routine
60 INPUT "enter file name of data file: ",F$
70 OPEN F$ FOR INPUT AS #1
80 FOR I=1 TO 5
90 FOR J=1 TO 24
100 INPUT #1,A(J,I),B(J,I)
110 NEXT J
115 NEXT I
120 CLOSE #1
150 FOR I=1 TO 5
160 FOR J=25 TO 48
170 A(J,I)=A(J-24,I)
175 B(J,I)=B(J-24,I)
180 NEXT J
190 NEXT I
200 REM calculate curves by summing up consumption
210 FOR I=1 TO 5
220 FOR J=1 TO 24
225 S(J,I)=0!
227 SMAX=0!
230 FOR N=1 TO 48-J
232 T(N,I)=0!
235 FOR K=1 TO J
240 T(N,I)=T(N,I)+A(K+N,I)
245 NEXT K
250 IF T(N,I)>SMAX THEN SMAX=T(N,I)
260 NEXT N
270 S(J,I)=SMAX
```

APPENDIX B

(Cont'd)

```
280 NEXT J
290 NEXT I
300 REM calculate recovery
310 FOR I=1 TO 5
320 FOR J=1 TO 24
325 SJ=J
330 R(J,I)=S(J,I)/SJ
340 NEXT J
350 NEXT I
351 INPUT "enter output file name: ",F$
352 OPEN F$ FOR OUTPUT AS #2
353 FOR I=1 TO 5
354 PRINT #2,24
355 FOR J=1 TO 48
356 WRITE #2, S(J,I), R(J,I)
357 NEXT J
358 NEXT I
359 CLOSE #2
360 GOSUB 1000
400 REM output recovery and storage requirements to a file
405 REM suitable for plotting by Energraphics.
406 FOUT$="D:\hoth2o\spiel.prn"
410 OPEN FOUT$ FOR OUTPUT AS #1
416 PRINT #1,5
417 PRINT #1,"I"
420 FOR I=1 TO 5
425 PRINT #1,24
430 FOR J=1 TO 24
440 WRITE #1,X1(J,I),Y1(J,I)
450 NEXT J
460 NEXT I
470 CLOSE #1
490 END
1000 REM subroutine to calculate spielvogel curves
1010 FOR I=1 TO 5
1015 A2=-R(1,I)/S(1,I):B2=R(1,I):X1(1,I)=0:Y1(1,I)=B2
1020 FOR J=2 TO 24
1022 OLDY=0!
1025 FOR K= J TO 24
1030 A1=-R(K,I)/S(K,I): B1=R(K,I)
1040 X=(B2-B1)/(A1-A2)
1050 Y=A1*X+B1
1055 IF Y>OLDY THEN N=K: X1(J,I)=X:Y1(J,I)=Y:OLDY=Y
1060 IF I=2 THEN LPRINT J,K,N,Y,OLDY
```

APPENDIX B
(Cont'd)

1070 NEXT K

1072 J=N

1075 A2=-R(J,I)/S(J,I):B2=R(J,I)

1080 NEXT J

1090 NEXT I

1100 RETURN

DISTRIBUTION

Dr. D. Mills
Mr. J.M. Bell
Mr. M. Perlman
Mr. N. Milligan (3)
Mr. J.G. Cassan
Dr. A.L. Punhani
Mrs. T.C. Olivares (2)

Director of Research
Electrical Research
Electrical Research
Electrical Research
Operations Research
Operations Research
Operations Research

Public Reference Centre
Editorial Section (2)
Office Copy (3)
Extra copy (3)